BITS and BYTES- Exercises solutions

1.

a) 4 numbers can be represented with 2 bits, 8 with 3 bits, 16 with 4 bits.

b) With each additional bit, the number of numbers that can be represented is multiplied by 2, so 5 bits can represent 32 numbers. We notice the pattern :

2 bits → 4 = 2^2 numbers

3 bits → 8 = 2^3 numbers

4 bits → 16 = 2^4 numbers

5 bits → 32 = 2^5 numbers

8 bits → 2^8 = 256 numbers

n bits → 2^n numbers

8 bits can represent 256 numbers and n bits can represent 2^n numbers.

2. The binary representations of odd numbers end with a 1 and that of even numbers end with a 0.

3. In the right-most column (unit column) the 0 and 1 alternate (0, 1, 0, 1, 0, 1…). In the next column (2's column) two 0's are followed by two 1's, followed by two 1's, etc. (0, 0, 1, 1, 0, 0, 1, 1, 0, 0, …). In the four's column the 0 and 1 alternate by block of 4 (0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0 ,1, 1, 1, 1…). Finally in the eights' columns the 0 and 1 alternate in block of eight (8 0's followed by 8 one, etc…)

4. The pattern above will continue.

Binary Number	Decimal Value
00000	0
00001	1
00010	2
00011	3
00100	4
00101	5
00110	6
00111	7
01000	8
01001	9
01010	10
01011	11
01100	12
01101	13
01110	14
01111	15

Binary Number	Decimal Value
10000	16
10001	17
10010	18
10011	19
10100	20
10101	21
10110	22
10111	23
11000	24
11001	25
11010	26
11011	27
11100	28
11101	29
11110	30
11111	31

5. Magic cards from 0 to 31.

1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31

2	3	6	7
10	11	14	15
18	19	22	23
26	27	30	31

4	5	6	7
12	13	14	15
20	21	22	23
28	29	30	31

8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31

16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31

6. Find the decimal values of the following binary numbers. (Note: to make them more readable we often write the digits of a binary numbers by groups of four.)
   a) $1000\ 0000_2$ = 128 (2^7 = 128 or use table below)
   b) $1111\ 1111_2$ → The number before `1 0000 0000` which is 2^8 = 256. So `1111 111` is 255. Or use table below.
   c) $0010\ 1110_2$ = `46(from table below: 32 +8 + 4 + 2 = 46)`
   d) $1011\ 0000_2$ = `176 (from table below: 128 + 32 + 16 = 176)`

	128	**64**	**32**	**16**	**8**	**4**	**2**	**1**
a)	1	0	0	0	0	0	0	0
b)	1	1	1	1	1	1	1	1
c)	0	0	1	0	1	1	1	0
d)	1	0	1	1	0	0	0	0

7.
   a) 64 = 2^6 so 64 = $100\ 0000_2$
   b) 65 = 64 + 1 so 65 = $100\ 0001_2$
   c) 43 43 – 32 = 11 11 – 8 = 3 3 – 2 = 1 1- 1 = 0 43 = $10\ 1011_2$
   d) 213 213 – 128 = 85 85 – 64 = 21 21 – 16 = 5 5 – 4 = 1 1 – 1 = 0 so 213 = $1101\ 0101_2$

	128	**64**	**32**	**16**	**8**	**4**	**2**	**1**
64				1				1
65				1		1		
43			1		1		1	1
213	1	1		1		1		1

8. 2^8 = 256 nonnegative integers can be represented with a byte. The largest integer that can be written with one byte is $1111\ 1111_2$ = 255.

9. Convert the following hexadecimal numbers to decimals.
   a) $3A_{16} = 3\cdot 16 + 10 = 58$

b) $612_{16} = 6 \cdot 16^2 + 1 \cdot 16 + 2 = 1554$

c) $FEB2_{16} = 15 \cdot 16^3 + 14 \cdot 16^2 + 11 \cdot 16 + 2 = 65{,}202$

10. Convert the following binary numbers to hexadecimals.

a) 101100101= 0001 0110 0101 = 165_{16}

b) 10001001111 = 0100 0100 1111 = $44F_{16}$

11. Convert the hexadecimal numbers from question 9 . to binary.

a) $3A_{16}$ = $0011\ 1010_2$

b) 612_{16} = $0110\ 0001\ 0010_2$

c) $FEB2_{16}$ = $1111\ 1110\ 1011\ 0010_2$

Exercises

12. Write the following numbers in scientific notation.
    a. $231 = 2.31 \cdot 10^2$
    b. $34.212 = 3.4212 \cdot 10$
    c. $-3409992 = -3.409992 \cdot 10^6$
    d. $0.01023 = 1.023 \cdot 10^{-2}$

13. Write the following numbers in their decimal form.
    a. $-9.86 \cdot 10^2 = -986$
    b. $1.24 \cdot 10^3 = 1{,}240$
    c. $6.9087 \cdot 10^{-5} = 0.000069087$
    d. – 8.349E+2 = -834.9

14. Give the 10-digit floating- point representations with a 6-digit mantissa and 3-digit exponent of the following numbers:
    a. 3892 → +389200-02
    b. .93452 → +934520-06
    c. 4,231,123 → +423112+01
    d. -0.0023012 → -230120-08

15. In this problem we consider the same10-digit floating- point representation as in question 16. Consider the number 0.234211.
    a. What is its floating-point representation?
       +234211-06
    b. What is the *smallest* number *larger* than 0.234211 that has a floating-point representation different from 0.234211 without loss of precision?
       (Hint: increase the mantissa you found in a) by 1 and convert the number back to decimal.)
       Add one to mantissa: +234212-06 →0.234212
    c. Give a number that would have same floating-point representation as 0.234211 and a number that would have same floating-point representation as the number you found in b). Choose the 2 numbers so that they are between 0.234211 and the number found in b).
       0.2342113 has the same representation as 0.234211 (replace the last 3 by any digit between 1 and 4 and it would work).
       0.2342116 has the same representation as 0.234212 (replace the 6 by any digit between 5 and 9 and it would work).
       The two given numbers are between 0.234211 and 0.234212.